Further Kinematics - Questions

June 2017 Mathematics Advanced Paper 1: Mechanics 1

1.

7. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two ships, P and Q, are moving with constant velocities.
The velocity of P is $(9\mathbf{i} - 2\mathbf{j})$ km h^{-1} and the velocity of Q is $(4\mathbf{i} + 8\mathbf{j})$ km h^{-1}

(*a*) Find the direction of motion of P, giving your answer as a bearing to the nearest degree.

(3)

When $t = 0$, the position vector of P is $(9\mathbf{i} + 10\mathbf{j})$ km and the position vector of Q is $(\mathbf{i} + 4\mathbf{j})$ km. At time t hours, the position vectors of P and Q are $\mathbf{p}$ km and $\mathbf{q}$ km respectively.

(*b*) Find an expression for

(i) $\mathbf{p}$ in terms of t,

(ii) $\mathbf{q}$ in terms of t.

(3)

(*c*) Hence show that, at time t hours,

$$\overrightarrow{QP} = (8 + 5t)\mathbf{i} + (6 - 10t)\mathbf{j}$$

(2)

(*d*) Find the values of t when the ships are 10 km apart.

(6)

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2.

1. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two cars P and Q are moving on straight horizontal roads with constant velocities. The velocity of P is $(15\mathbf{i} + 20\mathbf{j})$ m s^{-1} and the velocity of Q is $(20\mathbf{i} - 5\mathbf{j})$ m s^{-1}

(a) Find the direction of motion of Q, giving your answer as a bearing to the nearest degree.

(3)

At time $t = 0$, the position vector of P is $400\mathbf{i}$ metres and the position vector of Q is $800\mathbf{j}$ metres. At time t seconds, the position vectors of P and Q are $\mathbf{p}$ metres and $\mathbf{q}$ metres respectively.

(b) Find an expression for

(i) $\mathbf{p}$ in terms of t,

(ii) $\mathbf{q}$ in terms of t.

(3)

(c) Find the position vector of Q when Q is due west of P.

(4)

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3.

6. A particle P is moving with constant velocity. The position vector of P at time t seconds ($t \geq 0$) is $\mathbf{r}$ metres, relative to a fixed origin O, and is given by

$$\mathbf{r} = (2t - 3)\mathbf{i} + (4 - 5t)\mathbf{j}.$$

(a) Find the initial position vector of P.

(1)

The particle P passes through the point with position vector $(3.4\mathbf{i} - 12\mathbf{j})$ m at time T seconds.

(b) Find the value of T.

(3)

(c) Find the speed of P.

(4)

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4.

5. A particle P of mass 0.5 kg is moving under the action of a single force $(3\mathbf{i} - 2\mathbf{j})$ N.

(a) Show that the magnitude of the acceleration of P is $2\sqrt{13}$m s^{-2}.

(4)

At time $t = 0$, the velocity of P is $(\mathbf{i} + 3\mathbf{j})$ m s^{-1}.

(b) Find the velocity of P at time $t = 2$ seconds.

(3)

Another particle Q moves with constant velocity $\mathbf{v} = (2\mathbf{i} - \mathbf{j})$ m s^{-1}.

(c) Find the distance moved by Q in 2 seconds.

(2)

(d) Show that at time $t = 3.5$ seconds both particles are moving in the same direction.

(3)

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5.

7. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively.*]

The velocity, $\mathbf{v}$ m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}.$$

(a) Find the speed of P when $t = 0$.

(3)

(b) Find the bearing on which P is moving when $t = 2$.

(2)

(c) Find the value of t when P is moving

(i) parallel to **j**,

(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)

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6.

6. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.*]

A ship sets sail at 9 a.m. from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 a.m. the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(*a*) Find the speed of the ship in km h^{-1}.

(4)

(*b*) Show that the position vector $\mathbf{r}$ km of the ship, t hours after 9 a.m., is given by

$$\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}.$$

(2)

At 10 a.m. a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 a.m. the passenger observes that L is now south-west of the ship.

(*c*) Find the position vector of L.

(5)

May 2012 Mathematics Advanced Paper 1: Mechanics 1

7.

6. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving with constant velocity $(-12\mathbf{i} + 7.5\mathbf{j})$ km h^{-1}.

(*a*) Find the direction in which S is moving, giving your answer as a bearing.

(3)

At time t hours after noon, the position vector of S is $\mathbf{s}$ km. When $t = 0$, $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$.

(*b*) Write down $\mathbf{s}$ in terms of t.

(2)

A fixed beacon B is at the point with position vector $(7\mathbf{i} + 12.5\mathbf{j})$ km.

(*c*) Find the distance of S from B when $t = 3$.

(4)

(*d*) Find the distance of S from B when S is due north of B.

(4)

8.

7. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.*]

A boat P is moving with constant velocity $(-4\mathbf{i} + 8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i} - 8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i} + 8\mathbf{j}).$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

9.

7. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.*]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}.

(*a*) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 p.m., ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 p.m., the position vector of P is $\mathbf{p}$ km and the position vector of Q is $\mathbf{q}$ km.

(*b*) Write down expressions, in terms of t, for

(i) $\mathbf{p}$,

(ii) $\mathbf{q}$,

(iii) $\overrightarrow{PQ}$.

(5)

(*c*) Find the time when

(i) Q is due north of P,

(ii) Q is north-west of P.

(4)

Jan 2011 Mathematics Advanced Paper 1: Mechanics 1

10.

4. A particle P of mass 2 kg is moving under the action of a constant force $\mathbf{F}$ newtons. The velocity of P is $(2\mathbf{i} - 5\mathbf{j})$ m s^{-1} at time $t = 0$, and $(7\mathbf{i} + 10\mathbf{j})$ m s^{-1} at time $t = 5$ s.

Find

(a) the speed of P at $t = 0$,

(2)

(b) the vector $\mathbf{F}$ in the form $a\mathbf{i} + b\mathbf{j}$,

(5)

(c) the value of t when P is moving parallel to $\mathbf{i}$.

(4)

May 2010 Mathematics Advanced Paper 1: Mechanics 1

11.

1. A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. At time $t = 6$ s, P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j})$ m. Find the distance of P from the origin at time $t = 2$ s.

(5)

Jan 2010 Mathematics Advanced Paper 1: Mechanics 1

12.

7. [*In this question,* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is $\mathbf{s}$ km. When $t = 0$, $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j}$. When $t = 4$, $\mathbf{s} = 21\mathbf{i} + 10\mathbf{j}$. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that $\mathbf{s} = (3t + 9)\,\mathbf{i} + (4t - 6)\,\mathbf{j}$.

(2)

A lighthouse L is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km. When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

(6)